Turbulence models in *Code_Saturne*

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Outline

Boussinesq approximation

The turbulence viscosity approximation introduced by Boussinesq in 1877 states that the deviatoric Reynolds stress is proportional to the mean rate of strain, that is:

$$-\rho \left\langle u_i' u_j' \right\rangle + \frac{2}{3}\rho k \delta_{ij} = \rho \nu_t \left(\frac{\partial \left\langle U_i \right\rangle}{\partial x_j} + \frac{\partial \left\langle U_j \right\rangle}{\partial x_i} \right) \quad (1)$$

- Widely used, only need to find an expression for a scalar ν_t.
- Isotropic formulation, assumes homogeneity.
- Solid boundaries introduce anisotropy!

Zero and one equation models

- Determine ν_t by prescribing a length scale l (Prandtl, 1925)

$$\nu_t = l^2 \left| \frac{du}{dy} \right| \tag{2}$$

l related to the flow thickness δ, round jet $l/\delta \approx 0.075$, plane jet $l/\delta \approx 0.09$ wall flows $l = \kappa y$

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- Calculate a transport equation for ν_t. The Spalart-Allmaras model uses a transport equation for the viscosity including eight closure coefficients and three damping functions.

Two-equation models

- Calculate the length scale as a ratio of two variables, usually the turbulent kinetic energy (k) and the dissipation (ε) or the rate of dissipation (ω). Solve transport equations for them.

$$k = \frac{1}{2}\langle u_i' u_j' \rangle \qquad \varepsilon_{ij} = -2\nu \left\langle \frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k} \right\rangle \qquad \omega \sim \frac{\varepsilon}{k} \qquad (4)$$

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- These are considered complete models, no need to to have prior knowledge of the flow.
- Transport equations are NOT exact, always there is the need to model another term therefore approximations still needed.

k equation

The equation for the turbulent kinetic energy can be derived from the Navier-Stokes equations by multiplying the fluctuating momentum equation by u'_j.

$$\frac{\partial k}{\partial t} + \langle U_j \rangle \frac{\partial k}{\partial x_j} = P_k - \varepsilon + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \quad (5)$$

with

$$P_{ij} = -\langle u'_i u'_k \rangle \frac{\partial \langle U_j \rangle}{\partial x_k} - \langle u'_j u'_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} \quad \text{modelled as} \quad P_k = 2\nu_t S_{ij} S_{ij} \quad (6)$$

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- Turbulent viscosity $\nu_t = C_\mu k^2 / \varepsilon$
- Constants calibrated to match SOME experiments (decaying turbulence, free shear flows ...)

Other frequently used models

- $k-\omega$ Similar to $k-\varepsilon$ but can be integrated all the way down to the wall (although erroneous profiles for turbulent variables). Easy to converge but free-stream dependent.

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- SST (Shear Stress Transport): Combines $k-\varepsilon$ and $k-\omega$ via empirical functions based on the distance to the wall.
- Spallart-Allmaras: Solves a transport equation for ν_t. Very empirical, tuned for aerodynamic applications.

... In *Code_Saturne*

Keyword in *Code_Saturne*: `iturb`

- 10: Mixing length.
  User needs to prescribe the reference length of the problem.
- 20: $k - \varepsilon$ (Jones and Launder, 1972).
- 21: $k - \varepsilon$ with linear production (Guimet and Laurence, 2002).
  Production ($P_k = 2\nu_t S_{ij} S_{ij}$) is limited to a linear dependency on S_{ij}. Important in impingement regions.
- 60: Shear Stress transport, SST (Menter 1994).
  Mixes $k - \omega$ near the wall and $k - \varepsilon$ far away. Also has a limiter on the turbulent viscosity.

... In *Code_Saturne*

- ideuch: Type of wall function used for the wall boundary conditions (if k is available, u_k can be calculated).
- igrake: Whether gravity should be taken into account in the production term.
- ikecou: Coupling of the source terms of $k - \varepsilon$.
- iclkep: Clipping of negative values.
- relaxv: Relaxation factors for each variable.

Reynolds Stress equation

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- From the Navier-Stokes equation:

$$\frac{\partial \langle u_i' u_j' \rangle}{\partial t} + \langle U_k \rangle \frac{\partial \langle u_i' u_j' \rangle}{\partial x_k} = D_{ij}^{\nu} + D_{ij}^{T} + \phi_{ij} + P_{ij} + \varepsilon_{ij} \quad (8)$$

$$\mathbf{D}_{\mathbf{ij}}^{\nu} = \nu \frac{\partial^2 \langle u_i' u_j' \rangle}{\partial x_k \partial x_k} \; \mathbf{P}_{\mathbf{ij}} = -\langle u_i' u_k' \rangle \frac{\partial \langle U_j \rangle}{\partial x_k} - \langle u_j' u_k' \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} \quad (9)$$

$$D_{ij}^{T} = -\frac{\partial}{\partial x_k} \left(\langle u_i' u_j' u_k' \rangle + \left\langle \frac{p'}{\rho} u_j' \right\rangle \delta_{ik} + \left\langle \frac{p'}{\rho} u_i' \right\rangle \delta_{jk} \right) \quad (10)$$

$$\phi_{ij} = -\frac{1}{\rho} \left\langle p' \frac{\partial u_i'}{\partial x_j} \right\rangle - \frac{1}{\rho} \left\langle p' \frac{\partial u_j'}{\partial x_i} \right\rangle \qquad \varepsilon_{ij} = -2\nu \left\langle \frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k} \right\rangle \quad (11)$$

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- LRR (Launder et. al, 1975) "Isotropization of Production" + Return to isotropy (Rotta)

$$\phi_{ij} = -c_2 \left(P_{ij} - \frac{2}{3} P_k \delta_{ij} \right) - c_1 \frac{\varepsilon}{k} \left(\langle u_i' u_j' \rangle - \frac{2}{3} k \delta_{ij} \right) \quad (12)$$

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- More complex models (SSG, quadratic, cubic ...etc)
- Not treated the viscous sublayer yet!!

... In *Code_Saturne*

Keyword in *Code_Saturne*: `iturb`

- 30: Launder, Reece and Rodi, LLR (Launder et. al, 1975)
  Diffusion term modelled by GGDH.
- 31: Speziale, Sarkar and Gatski, SSG (Speziale et. al, 1991)
  Diffusion term modelled by SGDH.

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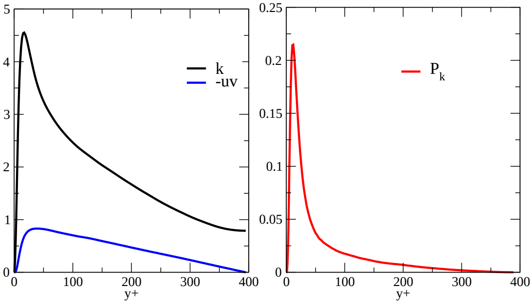
- The wall normal fluctuations are reduced therefore reducing mixing.

The wall effects

- No-slip: The boundary condition on the mean velocities creates large gradients where the turbulent production originates.



The wall effects

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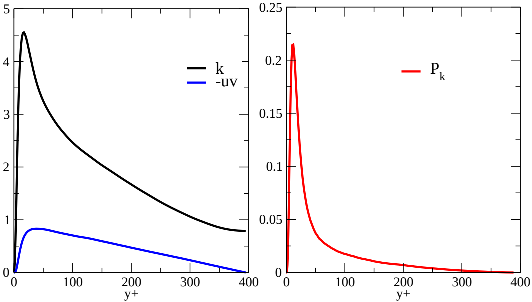
- Low Reynolds number effects: Interaction between energetic and dissipative scales.



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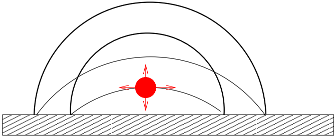
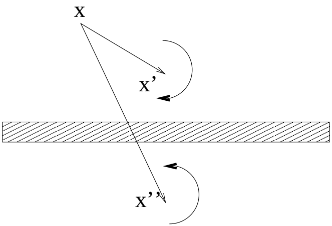
- Blocking effect: The impermeability condition affects the flow by adjusting the pressure field to ensure the incompressibility condition.





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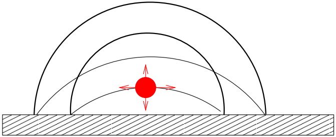
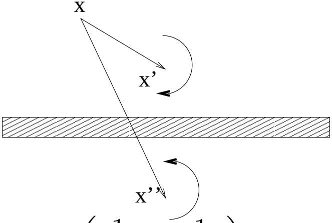
- Wall echo: Image term in Green's function at the other side of the wall produces an increase in the pressure.



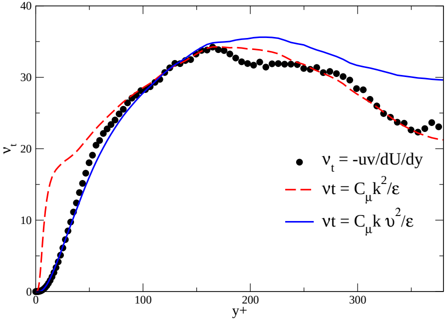


$$p^i(\mathbf{x},t) = -\frac{1}{4\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{\infty}^{\infty} S(\mathbf{x}',t)\left(\frac{1}{|\mathbf{r}'|}+\frac{1}{|\mathbf{r}''|}\right)d\mathbf{x}'$$

The $\overline{v^2} - f$ model

In order the simplify the RSM, the elliptic relaxation is introduced to the eddy viscosity approximation (Durbin,1995).

- Use of correct velocity scale near the wall, $\nu_t = C_\mu \overline{v^2} T$



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- Stiffness of the boundary condition makes it necessary to solve $\overline{v^2} - f$ coupled.

... In *Code_Saturne*

Keyword in *Code_Saturne*: `iturb=50`.
Instead of solving $\overline{v^2}$, a transport equation is solved for the ratio $\varphi = \overline{v^2}/k$ (Laurence et al., 2004):

$$\frac{D(\overline{v^2}/k)}{Dt} = f - \frac{(\overline{v^2}/k)}{k}\mathcal{P} + \frac{\partial}{\partial x_k}\left[\left(\nu + \frac{\nu_t}{\sigma_{(\overline{v^2}/k)}}\right)\frac{\partial(\overline{v^2}/k)}{\partial x_k}\right] + X$$

Where X is the "cross diffusion" term from the transformation:

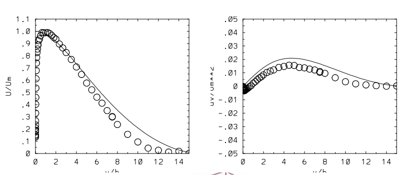
$$X = \frac{2}{k}\left(\nu + \frac{\nu_t}{\sigma_{(\overline{v^2}/k)}}\right)\frac{\partial(\overline{v^2}/k)}{\partial x_k}\frac{\partial k}{\partial x_k}$$

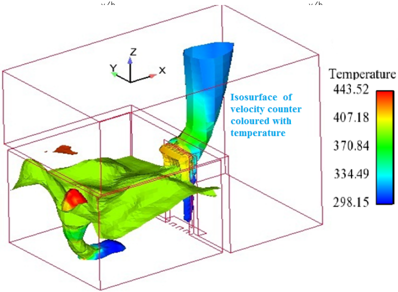
Which one to choose?

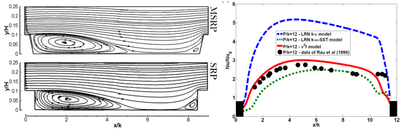
... It depends on the case.

EVMs

- Fast and robust.
- Can be used with wall functions to save CPU.
- Simple and easy to understand.
- Basic assumptions.
- Do not take into account anisotropy.
- limitations in flows with impingement, rotation, curvature, separation ...





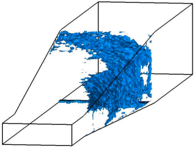


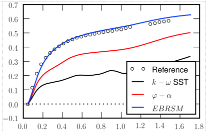
Which one to choose?

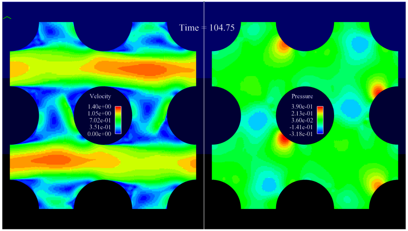
... and on the CPU available.

SMCs

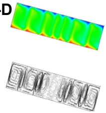
- More physics involved.
- Full anisotropic model.
- Exact production term.
- Better for 3D and unsteady flows.
- More equations $(u, v, w, p, \overline{u_i u_j}, \varepsilon)$
- More CPU.
- Can have convergence difficulties.



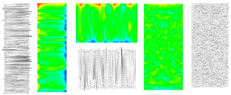




2-D



3-D



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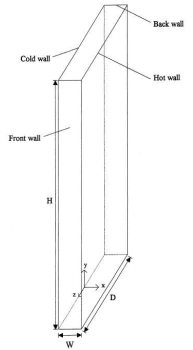
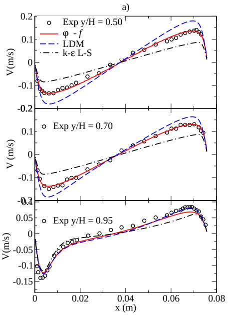
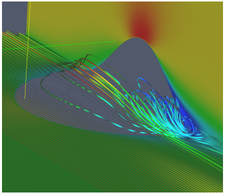
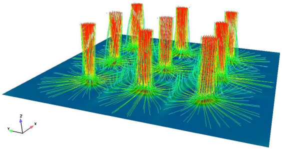
Low-Re models

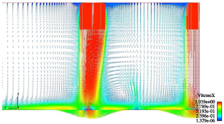
- Near wall region is important.
- Where friction coefficient or heat transfer are not in the log-law.
- Separated flows.
- Wall induced anisotropy.











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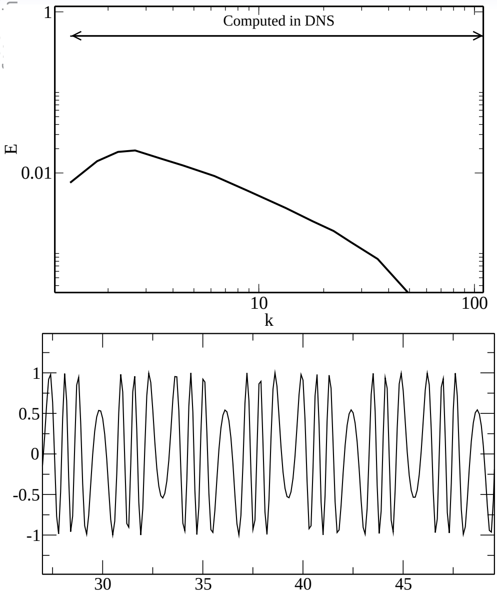

Turbulent flows simulations

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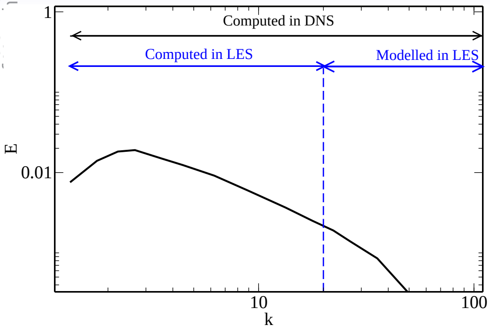

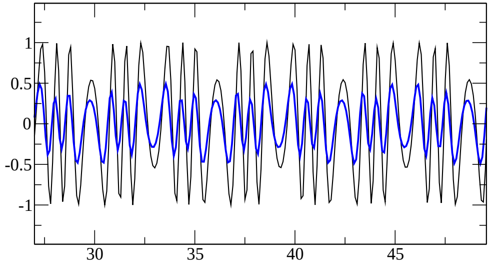

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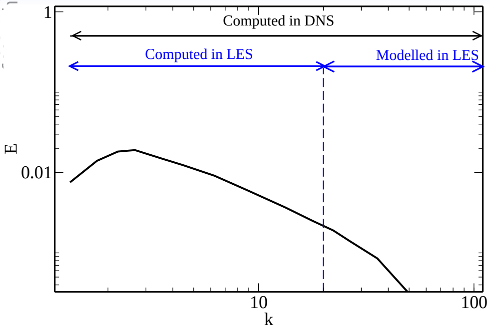

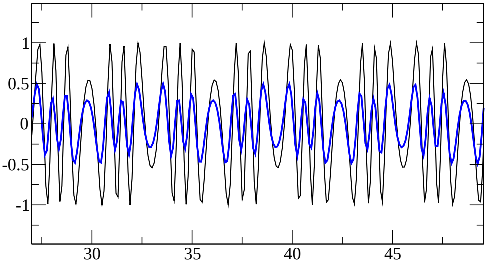

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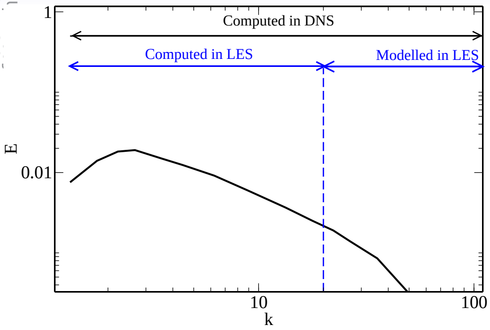

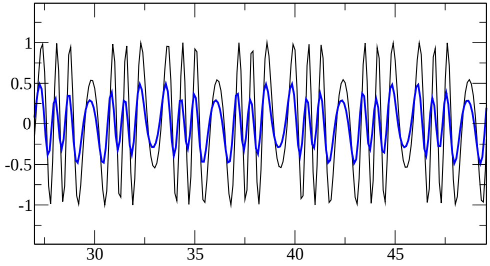

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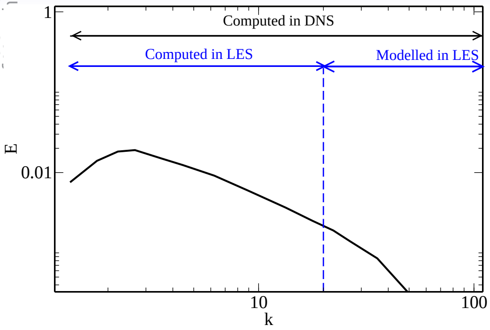

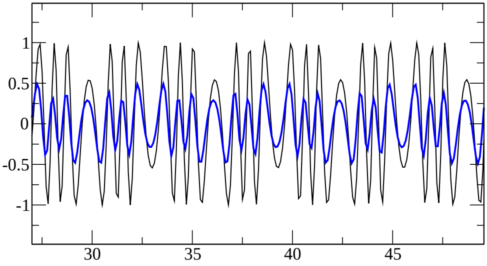

Turbulent flows simulations

- Take the instantaneous signal and filter it.
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- But they tend to be more homogeneous and easier to model.

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- Full 3D approach.

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- Requires very fine meshes near the solid boundaries.
- To get mean values, simulation needs to run long enough.

LES Modelling

By filtering the Navier-Stokes equations and unknown term arises:

$$\tau_{ij}^{R} = \overline{U_i U_j} - \overline{U_i}\ \overline{U_j} \tag{13}$$

- Assume all scales inside the filter width (Δ) are homogeneous.
- Homogeneous scales are easier to model.
- In practice, $\Delta = 2Vol^{1/3}$.
- Which means that to treat in-homogeneous regions Δ needs to be reduced.
- Smaller $\Delta \rightarrow$ smaller cells $\rightarrow$ higher number of cells needed.
- Classical example: Wall bounded flows.

The Smagorisnky (1964) model

Introduce a turbulent viscosity so that:

$$\tau_{ij} - \frac{2}{3}\tau_{kk}\delta_{ij} = 2\nu_t S_{ij} \tag{14}$$

with

$$\nu_t = (C_s\Delta)^2\sqrt{2S_{ij}S_{ij}} \quad \text{with} \quad S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{15}$$

Here C_s is a "constant". Theoretical value $C_s \approx 0.17$ but in reality it is adapted to the flow (e.g. Wall bounded flows use $C_s \approx 0.065$).

- ν_t doesn't vanish in laminar sublayer or transitional flows (and it should!).
- Van Driest damping is used in wall bounded flows $f_\mu = 1 - \exp(-y^+/A^+)$

Dynamic Smagorinsky (Germano, 1991)

Since the C_s is not constant, try to compute it dynamically for each flow.

- Use a second test filter and apply it to the filtered velocity field $U = \widetilde{\overline{U}} + (\overline{U} - \widetilde{\overline{U}}) - u'$
- Then compute form the velocity field: $\mathcal{L}_{ij} = \widetilde{\overline{U_i}\,\overline{U_j}} - \widetilde{\overline{U_i}}\,\widetilde{\overline{U_j}}$
- Compute $M_{ij} \equiv 2\overline{\Delta}^2 \widetilde{\overline{S}\,\overline{S_{ij}}} - 2\widetilde{\overline{\Delta}}^2 \widetilde{\overline{S}}\,\widetilde{\overline{S_{ij}}}$
- The mean-square error is minimised (Lilly 1992) by specifying: $c_s = M_{ij}L_{ij}/M_{kl}M_{kl}$

WALE model (Nicoud & Ducros, 1999)

Wall Adapting Local Eddy viscosity.

- Need to correct the near wall behaviour of the SGS models.
- Usually done by Van Driest damping but this requires y and u_τ
- Find a way to mimic the asymptotic behaviour:

$$\nu_t = (C_s\Delta)^2 \frac{(S^d_{ij}S^d_{ij})^{2/3}}{(S_{ij}S_{ij})^{5/2} + (S^d_{ij}S^d_{ij})^{5/4}} \tag{16}$$

with $S^d_{ij} = S_{ik}S_{kj} + \Omega_{ik}\Omega kj - \frac{1}{3}(S_{mn}S_{mn} - \Omega_{mn}\Omega_{mn})\delta_{ij}$

... In *Code_Saturne*

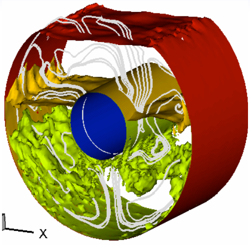
Keyword in *Code_Saturne*: `iturb`.

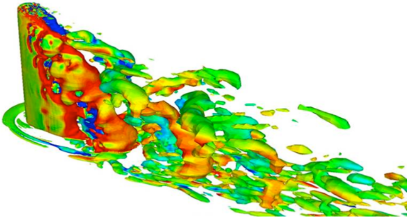
- 40: Smagorinsky. Thoroughly tested but C_s case dependant.
- 41: Dynamic. Useful when laminar regions are present (walls, transition, natural convection). Requires finer meshes (two filters). Negative ν_t might appear.
- 42: WALE. No need for wall damping. Correct asymptotic behaviour. Not very popular.

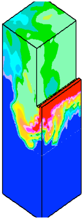
When using LES ...

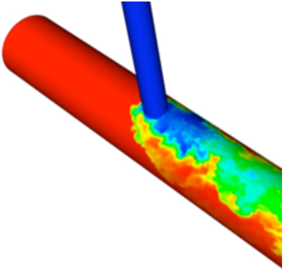
- Well resolved LES results often better than RANS but coarse LES worst than coarse RANS.
- Numerical issues are very important. Second order in time and space required.
- Need to extract statistical values to have any meaning.
- Meshing is very important. Need to know the scales in the flow so a precursor RANS simulation is very helpful.
- Cell distortion, high aspect ratio and excessive growth should be avoided.

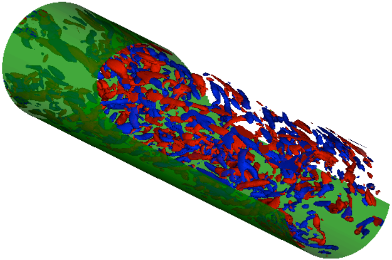
LES examples











Pictures from: Y.Addad, S. Benhamadouche and I. Afgan

Hybrid methods

When flow is too complex for RANS (EVMs or SMCs) and the mesh requirements for LES are too large ($\Delta y^+ \sim 1$, $\Delta x^+ \sim 50$, $\Delta z^+ \sim 20$).

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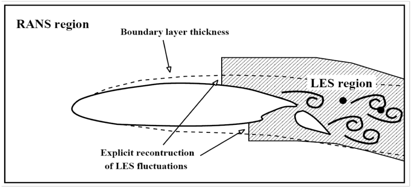
Interface

Prescribe an interface, one side RANS another LES.

- Needs to add turbulent information when going from RANS to LES.
- Good for streamwise coupling using synthetic turbulence.
- More difficult with wall normal coupling.



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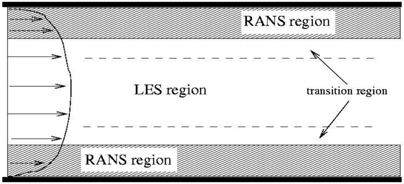


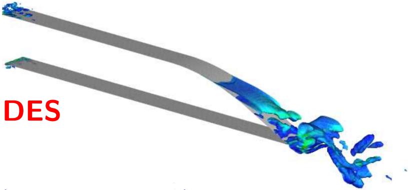
Seamless

Let the model change automatically.

- Needs a parameter to switch from RANS to LES, usually based on the cell size.
- Fluctuations can easily die while in RANS.
- User needs to carefully design the mesh for the appropriate switch to occur.

Hybrid approaches

- Detached Eddy Simulation (DES, Spalart, 2000)



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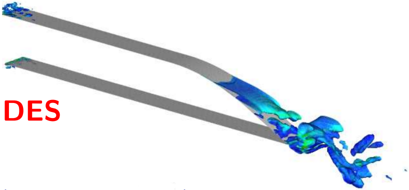

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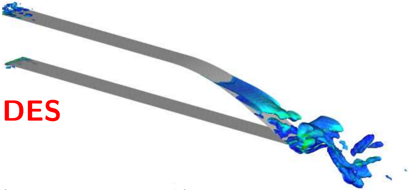

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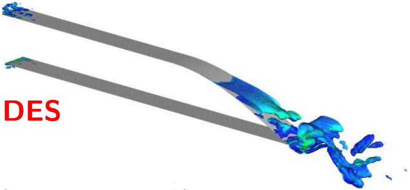

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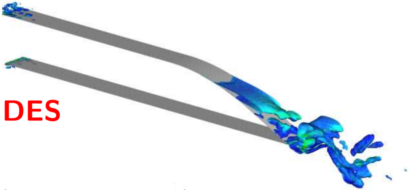

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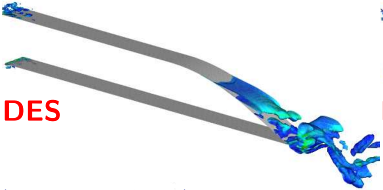

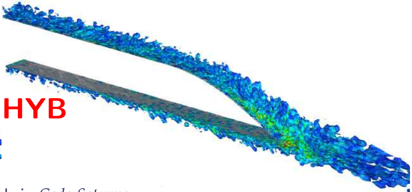

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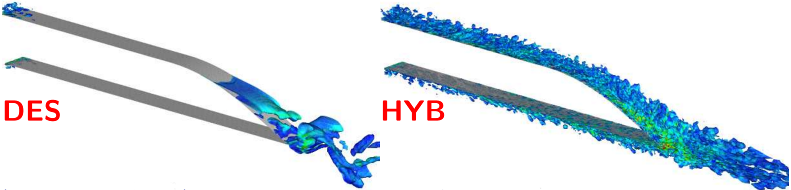

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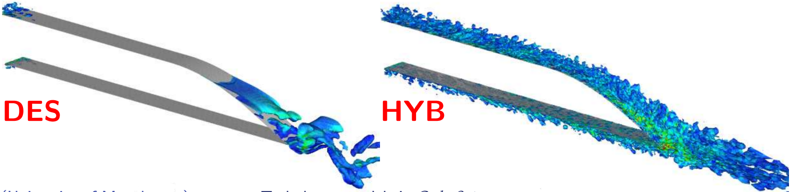

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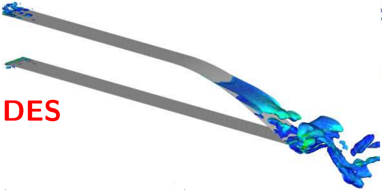

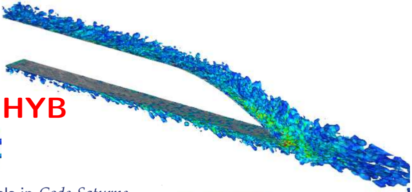

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  - Treat the fluctuating with LES (as intended).
  - Not as mesh dependent since both models act on the whole domain.